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EXAM COMPUTER VISION, INMCV-08

February 8, 2011, 9:00 hrs

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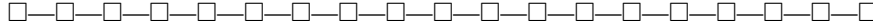
During the exam you may use the lab manual, copies of sheets, **provided they do not contain any notes**.

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. **Always motivate your answers.** *Good luck!*

Problem 1. (**2.0 pt**) Consider an image E with grey level distribution given by

$$E(u_0, v_0) = A \sin(\omega u_0) \tag{1}$$

at $t = 0$ (u_0 and v_0 denote position at $t = 0$), with ω the spatial frequency of the pattern, and A the amplitude. The motion field in the image plane is given by

$$\dot{u} = 1, \quad \dot{v} = 2. \tag{2}$$

a. (0.5 pt) Give an expression for $E(u, v, t)$ (**Hint:** first obtain expressions for $u(t)$ and $v(t)$).

b. (0.5 pt) Compute the observed temporal changes in irradiance $\frac{\partial E}{\partial t}$ as a function of u and v (**Hint:** if you did not solve part a., use the Horn-Schunck equation).

c. (1.0 pt) Given the temporal changes (optic flow) obtained in part b., can the motion field be recovered completely using the Horn-Schunck equation? If not, would addition of smoothness constraints solve this problem? Explain your answer.

Problem 2. (**2.5 pt**) Consider a stereo pair of images from two camera as shown below with $O_L = (-10, 0, 0)$ and $O_R = (10, 0, 0)$, $f = 20$
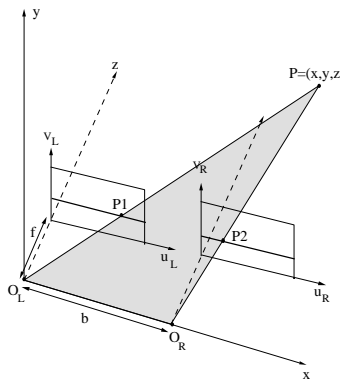

Figure 1: *Standard stereo set-up.*

a. (1 pt) Suppose a feature is detected in the left camera image at $(u_L, v_L) = (0, 0)$ and the right camera image at $(u_R, v_R) = (-2, 0)$. What is the (x, y, z)-position of the object?

b. (0.5 pt) Suppose two features are found at $(u_L, v_L) = (0, 0.01)$ $(u_R, v_R) = (-2, -0.02)$. Can these refer to the same object? Motivate your answer.

c. (1pt) Give two ways in which we can increase accuracy of a stereo set up.

Problem 3. (**2.5pt**) A linear scale space can be implemented as a convolution with Gaussian kernel

$$K_\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}. \tag{3}$$

of varying standard deviations σ. Given the following expression for a convolution of two Gaussian functions:

$$K_{\sigma_1}(x, y) * K_{\sigma_2}(x, y) = \frac{1}{2\pi(\sigma_1^2 + \sigma_2^2)} e^{-\frac{x^2+y^2}{2(\sigma_1^2+\sigma_2^2)}}, \tag{4}$$

and the equation

$$I_t(x, y) = \mathcal{S}^t I(x, y) = I_0(x, y) * K_{\sqrt{2t}}(x, y) \tag{5}$$

defines the scale space, with $I_0(x, y)$ the original image. Also, recall that a convolution with any kernel defines a linear, shift-invariant system. Consider the six axioms of scale spaces.

a. (**0.5 pt**) Argue that the linear scale space is translation invariant

b. (**0.5pt**) Argue that the linear scale space is rotation invariant

c. (**1.0pt**) Show that this scale space is causal in the sense of

$$\mathcal{S}^t \mathcal{S}^s I = \mathcal{S}^{t+s} I, \qquad \forall s, t \geq 0.$$

d. 0.5pt Show that this scale space is contrast invariant in the sense that

$$\mathcal{S}^t(gI) = g(\mathcal{S}^t I).$$

with g an arbitrary constant.

Problem 4. (**2.0 pt**) Consider a surface of the form

$$z = d + ax^2 + by,$$

a. (**1.5 pt**) Assuming a homogeneous texture with texture constant ρ_0, determine the observed texture density $\Gamma(u, v)$ under parallel projection ($u = x, v = y$).

b. (**0.5 pt** Assuming we observe the density $\Gamma(u, v)$ you derived. Would this allow exact reconstruction of the surface? If not: what is missing and what could we do to resolve the problem.